Digital Image Processing and Pattern Recognition

E1528

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Lecture 7

Order-statistic (Nonlinear) Filters

INSTRUCTOR

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Contents

- Introduction
- Median Filters
- Sharpening (High-pass) Spatial Filters
- Derivatives
- High-pass, Band-reject, and Bandpass Filters from Lowpass Filters
- Combining Spatial Enhancement Methods
- summary

Introduction

- Order-statistic filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the region encompassed by the filter.

- Smoothing is achieved by replacing the value of the center pixel with the value determined by the ranking result.

- The best-known filter in this category is the median filter, which, as its name implies, replaces the value of the center pixel by the median of the intensity values in the neighborhood of that pixel (the value of the center pixel is included in computing the median).

Median Filters

- Median filters provide excellent noise reduction capabilities for certain types of random noise, with considerably less blurring than linear smoothing filters of similar size.
- Median filters are particularly effective in the presence of impulse noise (sometimes called salt-and-pepper noise, when it manisfests itself as white and black dots superimposed on an image).
- The median, ξ, of a set of values is such that half the values in the set are less than or equal to ξ and half are greater than or equal to ξ.

Median Filters

- In order to perform median filtering at a point in an image, we first sort the values of the pixels in the neighborhood, determine their median, and assign that value to the pixel in the filtered image corresponding to the center of the neighborhood.
- For example, in a 3×3 neighborhood the median is the 5^{th} largest value, in a 5×5 neighborhood it is the 13^{th} largest value, and so on.

Median Filters

- When several values in a neighborhood are the same, all equal values are grouped.
- For example, suppose that a 3×3 neighborhood has values (10, 20, 20, 20, 15, 20, 20, 25, 100). These values are sorted as (10, 15, 20, 20, 20, 20, 20, 25, 100), which results in a median of 20. Thus, the principal function of median filters is to force points to be more like their neighbors.

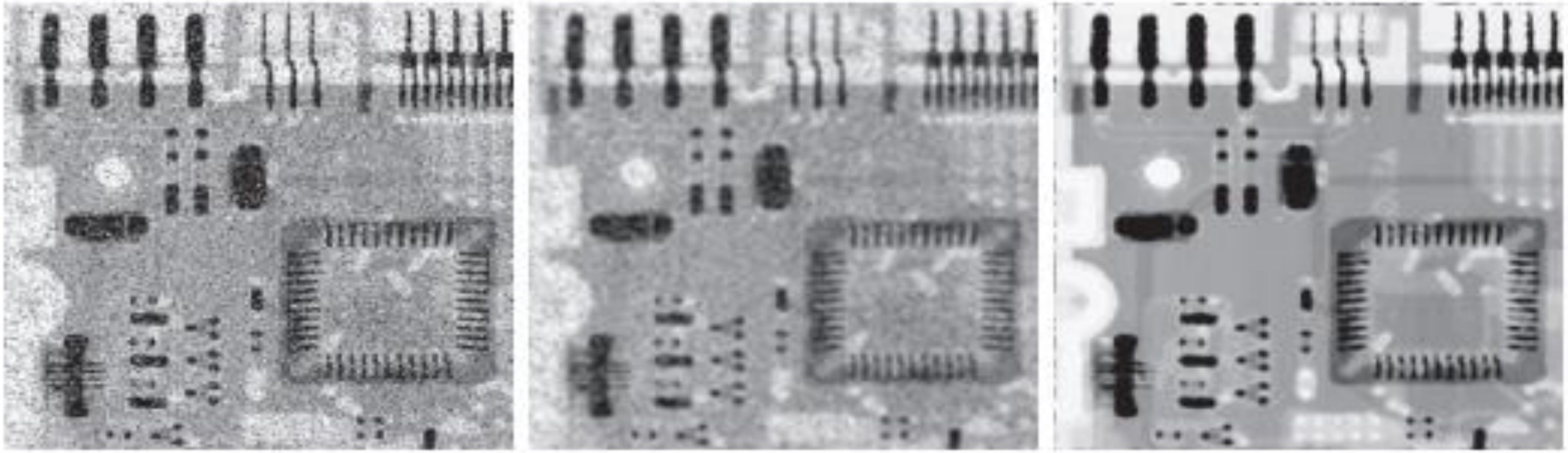
Median Filters

- The median filter is by far the most useful order-statistic filter in image processing but is not the only one.
- The median represents the 50^{th} percentile of a ranked set of numbers but ranking lends itself to many other possibilities.
- For example, using the 100^{th} percentile results in the so-called max filter, which is useful for finding the brightest points in an image or for eroding dark areas adjacent to light regions.

Median Filters

- The response of a 3×3 max filter is given by $R = \max \{ Z_k \mid k = 1, 2, 3, \ldots, 9\}$.
- the 0^{th} percentile filter is the min filter, used for the opposite purpose.
- Median, max, min, and several other nonlinear filters will be considered later.

➢ Median Filters


a b c

(a) X-ray image of a circuit board, corrupted by salt-and-pepper noise.

(b) Noise reduction using a 19 × 19 Gaussian lowpass filter kernel with **σ** = 3.

(c) Noise reduction using a 7 × 7 median filter.

Sharpening (High-pass) Spatial Filters

- Sharpening highlights transitions in intensity.
- Uses of image sharpening range from electronic printing and medical imaging to industrial inspection and autonomous guidance in military systems.
- last, we saw that image blurring could be accomplished in the spatial domain by pixel averaging (smoothing) in a neighborhood. Because averaging is analogous to integration, it is logical to conclude that sharpening can be accomplished by spatial differentiation.

Sharpening (High-pass) Spatial Filters

- In fact the following discussion deals with various ways of defining and implementing operators for sharpening by digital differentiation.
- The strength of the response of a derivative operator is proportional to the magnitude of the intensity discontinuity at the point at which the operator is applied.
- Thus, image differentiation enhances edges and other discontinuities (such as noise) and de-emphasizes areas with slowly varying intensities.

Sharpening (High-pass) Spatial Filters

- last, smoothing is often referred to as lowpass filtering, a term borrowed from frequency domain processing.
- In a similar manner, sharpening is often referred to as high-pass filtering.
- In this case, high frequencies (which are responsible for fine details) are passed, while low frequencies are attenuated or rejected.

Foundation

- In the next sections, we will consider in some detail sharpening filters that are based on first- and second-order derivatives, respectively.
- Before proceeding with that discussion, however, we stop to look at some of the fundamental properties of these derivatives in a digital context. To To simplify the explanation, we focus attention initially on one-dimensional derivatives.
- We are interested in the behavior of these derivatives in areas of constant intensity, at the onset and end of discontinuities (step and ramp discontinuities), and along intensity ramps.

Derivatives

- Derivatives of a digital function are defined in terms of differences.

- There are various ways to define these differences. However, we require that any definition we use for a first derivative:

  1. Must be zero in areas of constant intensity.
  2. Must be nonzero at the onset of an intensity step or ramp.
  3. Must be nonzero along intensity ramps.

- Similarly, any definition of a second derivative

  1. Must be zero in areas of constant intensity.
  2. Must be nonzero at the onset and end of an intensity step or ramp.
  3. Must be zero along intensity ramps.

Derivatives

- A basic definition of the first-order derivative of a one-dimensional function f (x) is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x) \qquad (1)$$

- We define the second-order derivative of f (x) as the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x) \qquad (2)$$

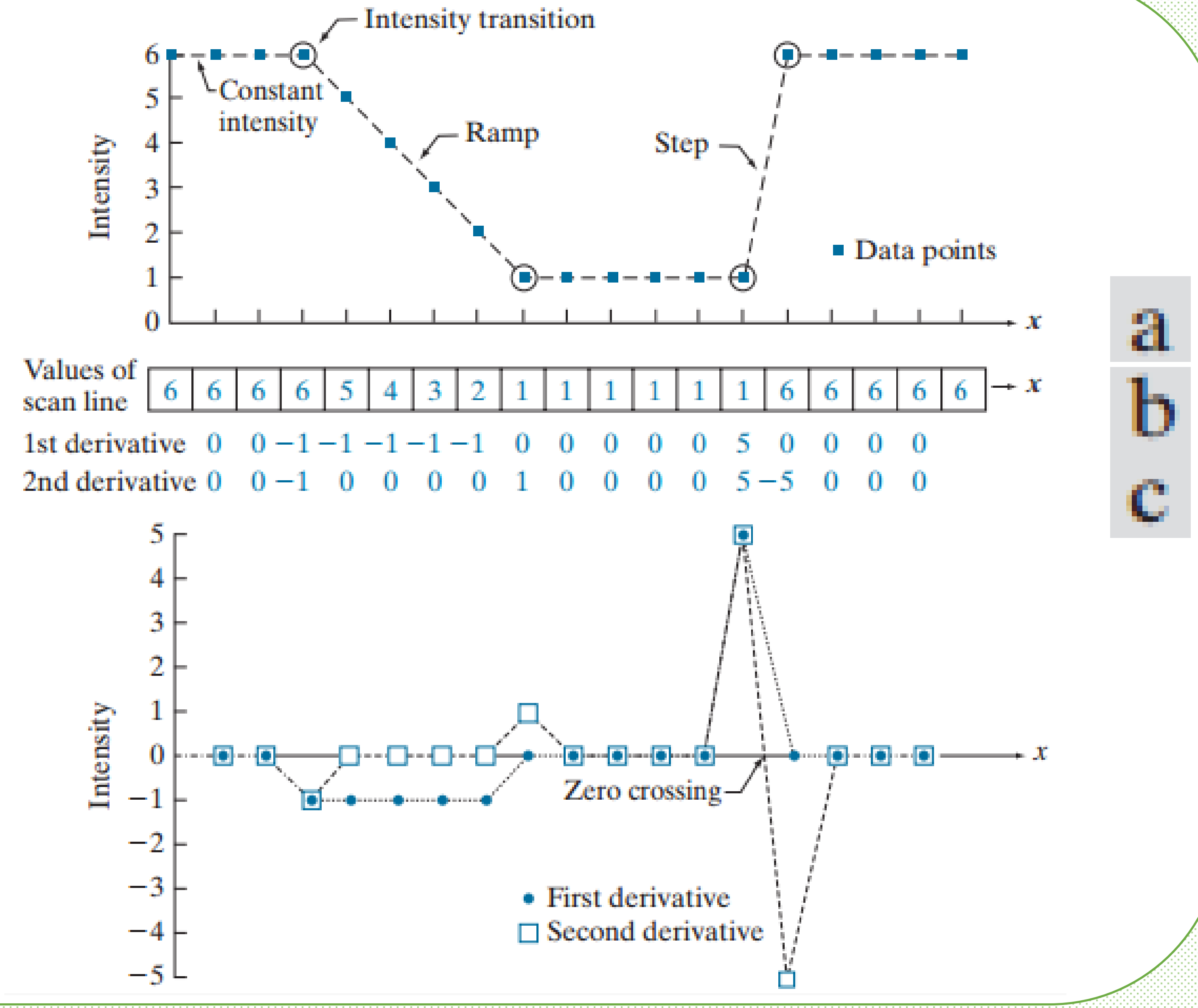
➢ Derivatives

(a) A section of a horizontal scan line from an image, showing ramp and step edges, as well as constant segments.

(b) Values of the scan line and its derivatives.

(c) Plot of the derivatives, showing a zero crossing.

In (a) and (c) points were joined by dashed lines as a visual aid.



Using The Second Derivative For Image Sharpening the Laplacian

- In this section we discuss the implementation of 2-D, second-order derivatives and their use for image sharpening.
- The approach consists of defining a discrete formulation of the second-order derivative and then constructing a filter kernel based on that formulation.
- we are interested here in isotropic kernels, whose response is independent of the direction of intensity discontinuities in the image to which the filter is applied.

➤ Using The Second Derivative For Image Sharpening the Laplacian

- It can be shown that the simplest isotropic derivative operator (kernel) is the Laplacian, which, for a function (image) f(x,y) of two variables, is defined as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \qquad (3)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y) \qquad (4)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y) \qquad (5)$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y) \qquad (6)$$

➢ Using The Second Derivative For Image Sharpening the Laplacian

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a b c d

(a) Laplacian kernel used to implement last Eq.

(b) Kernel used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other Laplacian kernels.

High-pass, Band-reject, and Bandpass Filters from Lowpass Filters

- Spatial and frequency-domain linear filters are classified into four broad categories: lowpass and high-pass filters, which we introduced, and bandpass and band-reject filters, which we introduce in this section.
- We mentioned at the beginning of last Section that the other three types of filters can be constructed from lowpass filters. In this section we explore methods for doing this.
- We know from earlier discussions that lowpass filters attenuate or delete high frequencies, while passing low frequencies.
- A high-pass filter behaves in exactly the opposite manner.

➢ High-pass, Band-reject, and Bandpass Filters from Lowpass Filters

Transfer functions of ideal 1-D filters in the frequency domain

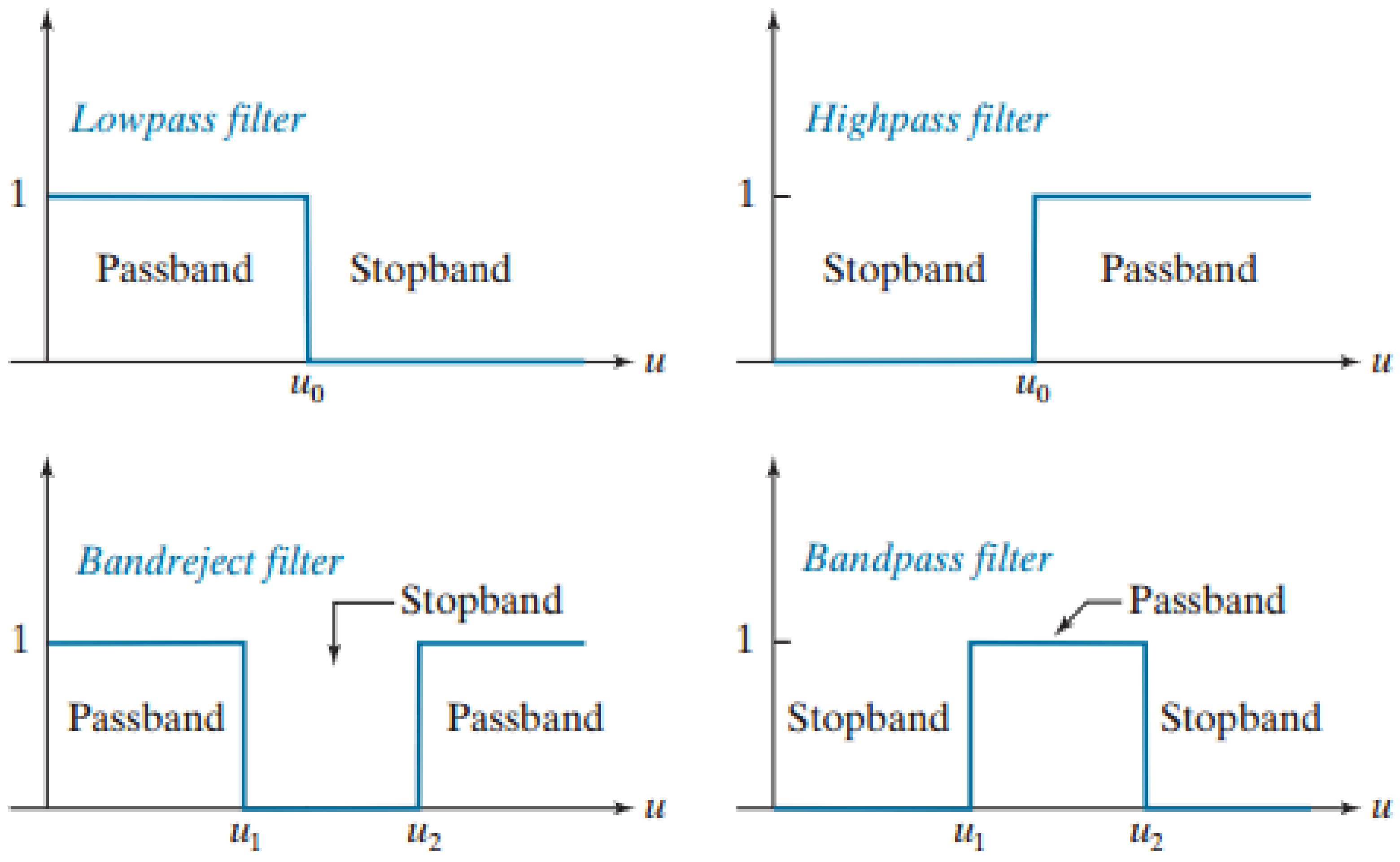
(u denotes frequency).

(a) Lowpass filter.

(b) High-pass filter.

(c) Band-reject filter.

(d) Bandpass filter.



High-pass, Band-reject, and Bandpass Filters from Lowpass Filters

- A high-pass filter transfer function is obtained by subtracting a lowpass function from 1. This operation is in the frequency domain.

- As you know, a constant in the frequency domain is an impulse in the spatial domain.

- Thus, we obtain a high-pass filter kernel in the spatial domain by subtracting a lowpass filter kernel from a unit impulse with the same center as the kernel. An image filtered with this kernel is the same as an image obtained by subtracting a lowpass-filtered image from the original image.

High-pass, Band-reject, and Bandpass Filters from Lowpass Filters

- Figure (c) shows the transfer function of a band-reject filter. This transfer function can be constructed from the sum of a lowpass and a high-pass function with different cut-off frequencies.
- The bandpass filter transfer function in Fig.(d) can be obtained by subtracting the band-reject function from 1 (a unit impulse in the spatial domain).
- Band-reject filters are also referred to as notch filters.

➢ High-pass, Band-reject, and Bandpass Filters from Lowpass Filters

Filter type	Spatial kernel in terms of lowpass kernel, *lp*
Lowpass	$lp(x,y)$
Highpass	$hp(x,y) = \delta(x,y) - lp(x,y)$
Bandreject	$br(x,y) = lp_1(x,y) + hp_2(x,y)$ $= lp_1(x,y) + [\delta(x,y) - lp_2(x,y)]$
Bandpass	$bp(x,y) = \delta(x,y) - br(x,y)$ $= \delta(x,y) - [lp_1(x,y) + [\delta(x,y) - lp_2(x,y)]]$

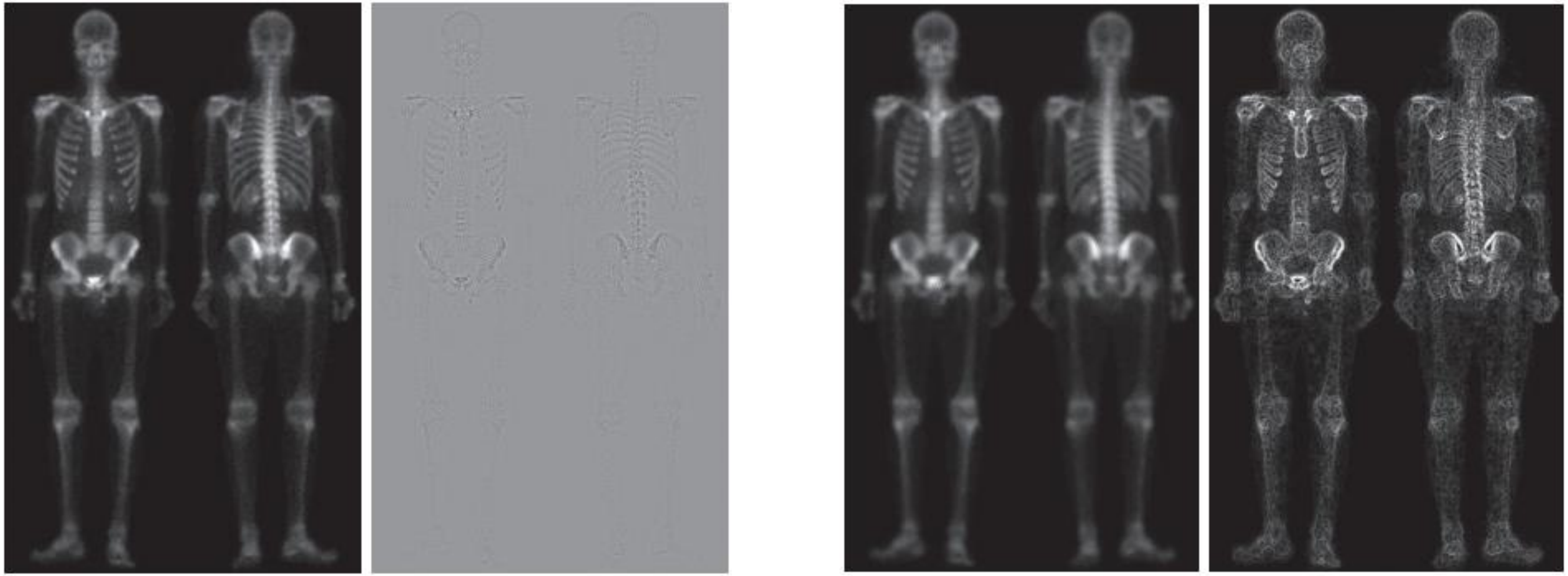
Combining Spatial Enhancement Methods

- we illustrate how to combine several of the approaches developed to address a difficult image enhancement task.
- The image in next Fig.(a) is a nuclear whole body bone scan, used to detect diseases such as bone infections and tumors.
- Our objective is to enhance this image by sharpening it and by bringing out more of the skeletal detail.
- The narrow dynamic range of the intensity levels and high noise content make this image difficult to enhance.

Combining Spatial Enhancement Methods

- The strategy we will follow is to utilize the Laplacian to highlight fine detail, and the gradient to enhance prominent edges.
- For reasons that will be explained shortly, a smoothed version of the gradient image will be used to mask the Laplacian image.
- Finally, we will attempt to increase the dynamic range of the intensity levels by using an intensity transformation.

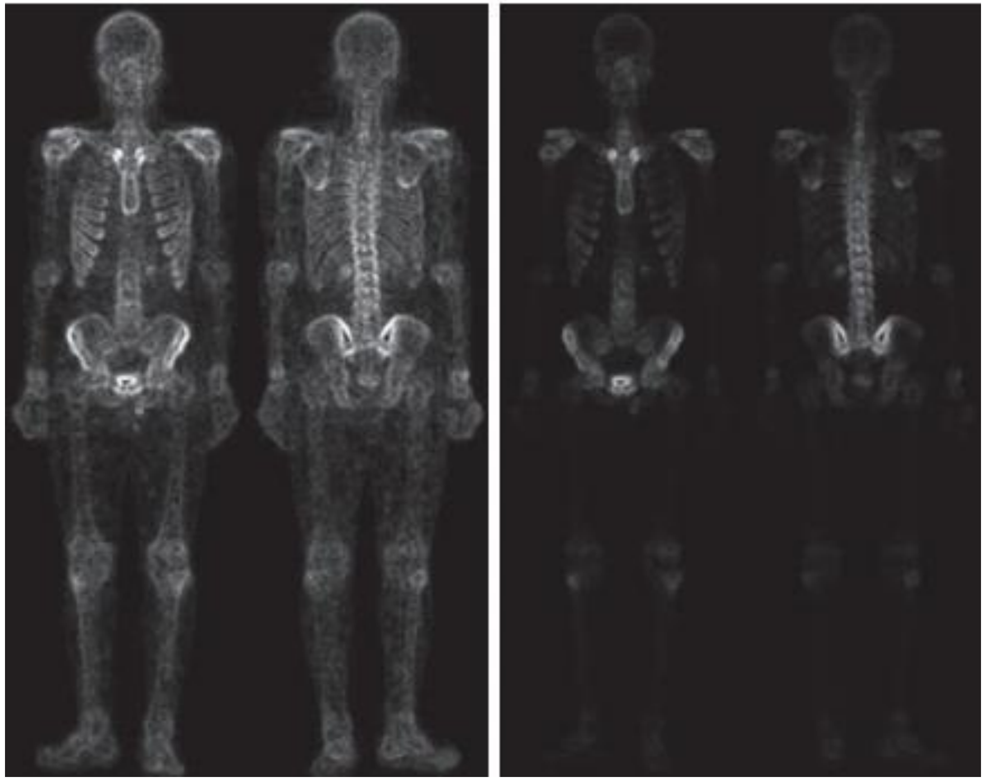
➢ Combining Spatial Enhancement Methods



a b c d

(a) Image of whole-body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of image (a).

➢ Combining Spatial Enhancement Methods



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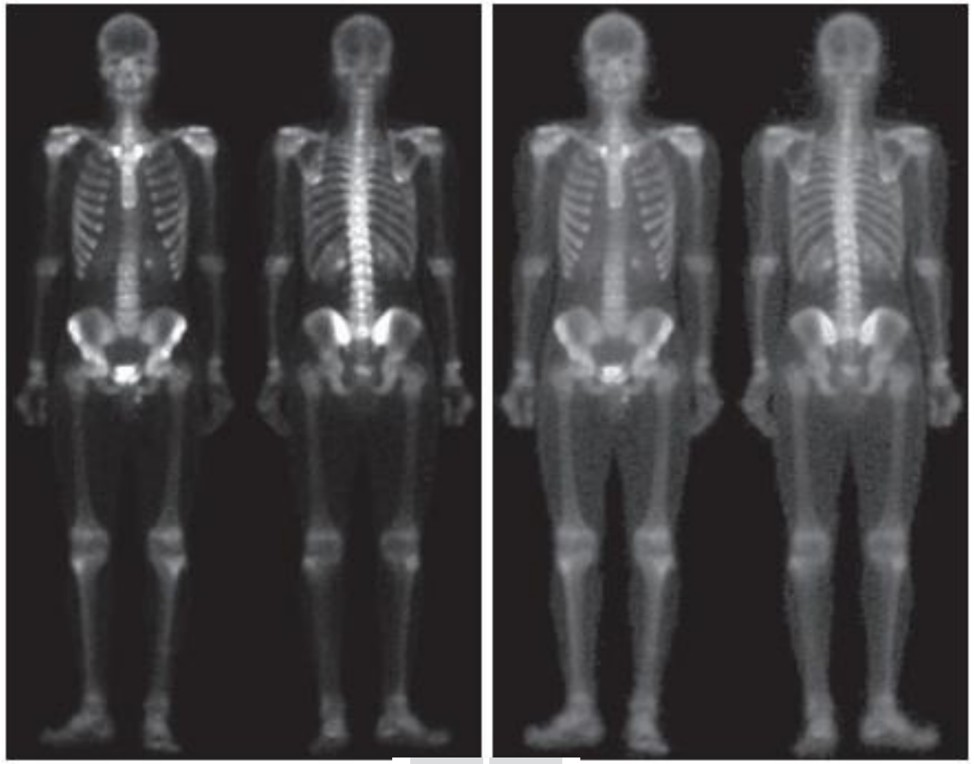

g h

(e) Sobel image smoothed with a 5 × 5 box filter. (f) Mask image formed by the product of (b) and (e). (g) Sharpened image obtained by the adding images (a) and (f). (h) Final result obtained by applying a powerlaw transformation to (g). Compare images (g) and (h) with (a).

Summary

- The material is representative of current techniques used for intensity transformations and spatial filtering.
- The topics were selected for their value as fundamental material that would serve as a foundation in an evolving field.
- Although most of the examples used in this chapter deal with image enhancement, the techniques presented are perfectly general, and you will encounter many of them again throughout the remaining chapters in contexts unrelated to enhancement.


Thank you